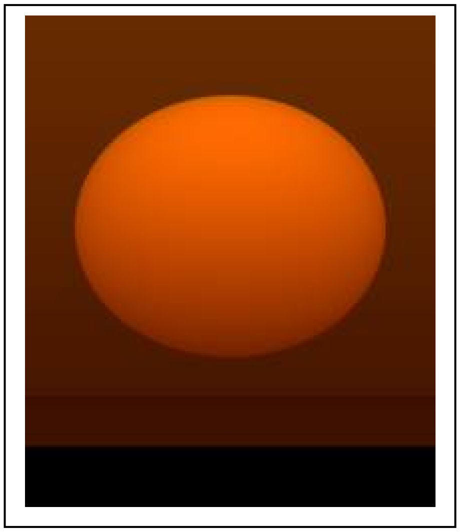
Three Mathematical Ways to Describe Light Extinction



When light passes through a medium it can lose some of its intensity. Scientists call this extinction. Depending on what properties they want to highlight in a calculation or a measurement, different ways of expressing extinction by a medium have arisen.

Opacity - Symbol τ : $I = I_0\, e^{-\tau}$

Decibels - Symbol D : $I = I_0\, 10^{-D/10}$

Extinction Coefficient - Symbol C : $I = I_0\, e^{-Cx}$

Problem 1 - If $e = 10^{0.434}$, and $10 = e^{2.3}$ write all three equations A) in base-10 B) in base-e.

Problem 2 – In base-10, what is the relationship between τ, D and C?

Problem 3 – In base-e, what is the relationship between τ, D and C?

Problem 4 - The SAGE III instrument measures a 1 Decibel (1 dB) drop in the sun's brightness along a path through the atmosphere of x=2000 km. What is the optical depth and extinction coefficient for this region of the atmosphere?

Problem 1 - If $e = 10^{0.434}$ and $10 = e^{2.3}$ write all three equations A) in base-10 B) in base-e.

A) $I = I_0 e^{-\tau}$ $I = I0 (10^{0.434})^{\tau}$ so $\mathbf{I = I_0 10^{-0.434\tau}}$

$I = I_0 10^{-D/10}$ unchanged so $\mathbf{I = I_0 10^{-D/10}}$

$I = I_0 e^{-Cx}$ $I = I_0 (10^{0.434})^{(-Cx)}$ so $\mathbf{I = I_0 10^{-0.434Cx}}$

B) $\mathbf{I = I_0 e^{-\tau}}$ unchanged

$I = I_0 (e^{2.3})^{-D/10}$ so $\mathbf{I = I_0 e^{-0.23D}}$

$\mathbf{I = I_0 e^{-Cx}}$ unchanged

Problem 2 – In base-10, what is the relationship between τ, D and C?

Answer: Just set the exponential factors equal to each other in Problem 1 A:

$-0.434\tau = -D/10 = -0.434Cx$ so after simplifying we get $\mathbf{\tau = 0.23D = Cx}$

Problem 3 – In base-e, what is the relationship between τ, D and C?

Answer: Set the exponential factors equal to each other in Problem 1 B: $\mathbf{\tau = 0.23D = Cx}$

Problem 4 - The SAGE III instrument measures a 1 Decibel (1 dB) drop in the sun's brightness along a path through the atmosphere of x=2000 km. What is the optical depth and extinction coefficient for this region of the atmosphere?

Answer: For 1 dB, and from Problem 2 (or 3!) we have

$\tau = 0.23$ D so
$\tau = 0.23 \times 1$ dB
$\tau =$ **0.23**.

For 1 dB and for x = 2000 km, we have
0.23 dB = Cx and so
0.23 = 2000C and so
C = 0.000115 km^{-1}.